Oblique vaults

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ABSTRACT: This research proposes the analysis of a spacious technical literature on the subject of the stone oblique vaults, retained the most refined theoretical speculation of the stereotomic lesson. The study, starting from the first medieval formulations of V. de Honnecourt, arrives at the analysis of the passages and biais passé technologies, passing through the important formulations of de l'Orme and the Spanish studies, arriving at the English and Italian studies of XIX century. Past attention has mainly focused on methods for building larger and more complex structures, dwelling on the main well-known types of equipment like the spiral and perpendicular methods, developed in the nineteenth century in France and England. The multidisciplinary appearance, typical of the nineteenth-century searches, agreed to move the field of analysis to more complex appearance like the mechanics, the mathematics and the geometry of the structures.

INTRODUCTION

The stone oblique vaults

During the eighteenth century the theme of the *ponts biais* involved eminent teachers of geometry and stereotomy, as well as mechanical and technical experts from the stone buildings: The input for this research was the significant increase in rail networks in Europe at the beginning of the century, which often posed professional workers with the need of having to cross inclined lines of the rail, just with an oblique bridge.
This requirement forced the engineers engaged in the design of rail infrastructure to deal with the appearance of geometric and tectonic cut stone, feeding an international debate on the issue of stone bridges.
Among the most significant achievements we could report at the time, there was the bridge on *Trilport Marne*, designed by *Hupeau* and *Chézy* between 1756 and 1760 (destroyed by a mine during the war in 1814), but above all the *Pont Neuf* in Paris (the oldest stone bridge in the capital, designed by *J. du Androuet Cerceau*, F. *des Isles, G. Marchand, T. Métezeau*, one of whose segments which consists of 5 arches applianced with orthogonal method) opened in 1603.
Despite significant examples in France, the first modern investigation on oblique bridges and related achievements were British studies, such as *Chapman* in Ireland and *Finlay* in 1787.
The practical initial speculation then became defining the most appropriate solution to the problem, outlining a geographical subdivision (France, Spain, England, Italy) based on the principal methods of equipment, whose supporters argued with detailed studies of the stereotomic treatises of the principles of geometry, the essays of applied mechanics.
This short essay will attempt to reconstruct, through the nineteenth sources, the process of the historical and academic subject of oblique bridges, from the roots of stereotomic treatises to modern publications.

OBLIQUE ARCHES AND VAULTS IN THE FIFTEENTH TO NINETEENTH CENTURIES

The oldest formulations around the theme of oblique vaults can be found in French stereotomic treatises from the sixteenth to seventeenth centuries: P. *de l'Orme, J.B. de la Rue, Frezier, P de la Hire., V. de Honnecourt*. However, it deals with "passages" of a small size that would result in the conformation of a single row of stones.

The first source is the oldest book of *Villard de Honnecourt* (1225): According to the structure of the book, there is an oblique drawing accompanied by a caption. Despite the lack of comments and theoretical formulations, the illustration places the knowledge of this type of structure among the knowledge of the Middle Ages.
Le premier tome de l'architecture (1567) by Philibert de l'Orme is a milestone for the study of stereotomy in step with the tradition of medieval maîtres - maçons to Renaissance architectural design.
The importance of the treatise, published in 1567, is extensive: *de l'Orme* is the first that wrote down the knowledge that until then was passed down orally, allowing more reflective epures of the trait and the Medieval models, inaugurating the possibility and potential of variation through excogitation, of the invention, evident then in Mannerism and the Baroque.

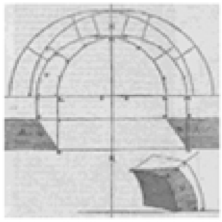
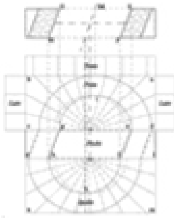
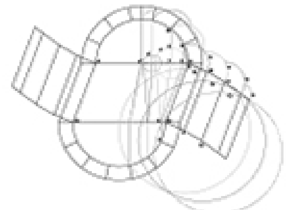
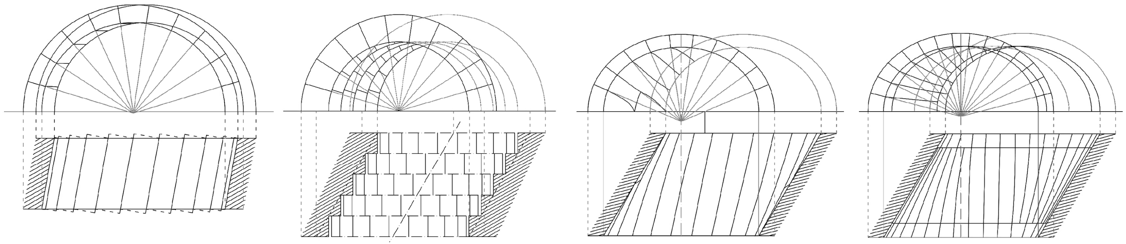




Figure 1: Oblique *biais passè,* built cutting the facade with orthogonal planes;(de l'Orme 1567, pp. 67 – 69)
Figure 2: Oblique arch; (J. Caramuel)
Figure 3: Development of surface, by the method of Vandelvira; (de Paola, Minenna's drawing from Vandelvira, *El tratado de Alonso de Vandelvira*)

Many of the solutions presented in the treatise are examples of *passages biais* as a connective between an old factory and a newly built one. The solutions proposed by *de l'Orme* became the rule on which the orthogonal equipment of bridges was built, strongly supported by French studies, generated by cutting the facade with orthogonal planes.
In *Architectura civil recta, y obliqua* (1678), *Caramuel L.Juan* devotes some articles to oblique arches, indicated by the geometric design. In *El Tratado de Arquitectura de Alonso de Vandelvira*, the epynomous author shows the construction reached with parallel joints, through a graphic construction, although not very clearly.
Martinez Gines de Aranda, a Spanish architect that lived at the turn of the sixteenth and seventeenth centuries, author of *Cerramientos y trazas de montéas* [1598 -1608], is the nucleus around which studies in the Spanish area will develop.
Despite the importance of these treatises in the transmission of constructive knowledge and stereotomic medieval themes of oblique structures, those were limited to *passages*: In this type of structure, given the limited size, the stones can be cut according to a parallel joint to the generation line of the cylindre, without excessive raise problems in terms of structural static. Bringing the same principles on a bridge, the study becomes more complex: here the static effects arise, known as trusts in the void.
The discussion of the topic, then, in subsequent treatises, it becomes more complex and has the support of different disciplines: stereotomy, descriptive geometry and solid geometry, mechanics and mathematics.
The equipment methods contained in subsequent treatises can be summarised as follows:

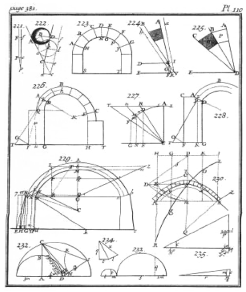


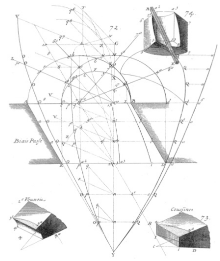
PARALLEL JOINTS PARALLEL ARCHES ELICOIDAL ORTHOGONAL

Figure 4: Concept of the main kinds of oblique bridges' equipments, with joint-projection on the plane; (Minenna, De Paola, 2008, *Ponti in pietra da taglio con apparecchiature stereotomiche*)

M. Graeff, (1897), explains details about twisted equipment, forming a reference point for those treated next.
The *Traité de la coupe des pierres* of *Philippe de la Hire* is the opening of stereotomy to the mechanics applied to buildings: La Hire was the first of the time thinking both as regards the geometry of the *coupe des pierres*, both determining the forces that in the second half of the seventeenth century will start a systematic theoretical discussion. La Hire introduces the principles by which to conduct the conformation of stones and joints, the behaviour of structures and the consequent possibility of breaking (45 degrees on the line of face, which was confirmed by Frézier for an angle of 22 -25 degrees), according to the monolitic stereotomic principle.
By the time of La Hire's death, a clear separation between mechanical and stereotomy was reaffirmed in the

treatise of Frézier, composed of three volumes dedicated to the problem of the coupe des pierres and an appendix on the mechanical behaviour of the vault. After *La Theorie et la Pratique de la Coupe des Pierres et des* Bois .., stereotomy, developed spontaneously in the *savoir faire* of the *appareilleurs*, becames "Science du Mathématicien"; a discipline based on geometry: Instead of the monolithic stone paradigm, the conformation of the vault is determined depending on the line taken by the pressures to which it was subject.
The eighteenth century was characterised by a rapid and extensive development of studies on the mechanics applied to architecture, accompanied by a progressive weakening of theoretical interest for stereotomy.
The only treatise that enjoyed widespread popularity was the one of *Jean Baptiste de la Rue*, even if entirely devoid of original studies.





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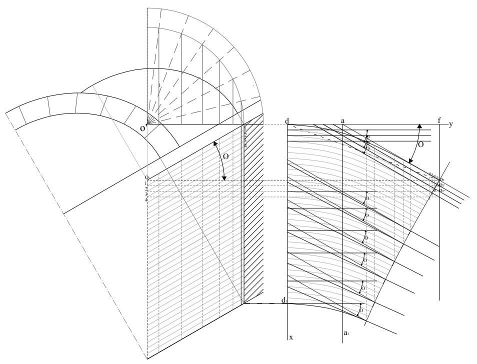
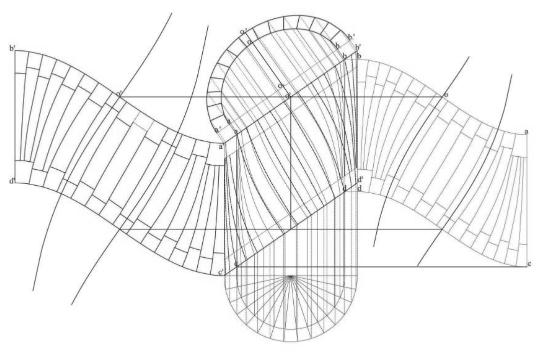
Figures 5 -6: Arch behaviour (left) and oblique arch trait; (*Frézier*, 1737, PL110)

Jean Chaix, work engineer director at the *Ecole Central*, is occupied mainly with the completion of rail networks from Paris to the Swiss border and from Tunis to the Algerian border, *chef de section* of the Compagnie des Chemins de Fer du Nord, addressing the design and construction of bridges.
In *Traité sur la coupe des pierres*, starts from the premise that the stereotomy was based on descriptive geometry's methods, founded by trait. In the second Volume of *Traité des Ponts* the theme of bridges with masonry oblique vaults brings the trait in various types of equipment that in previous years had been defined. Mechanical research about the compression is more precisely defined as well as the conformation of all joint in the geometric construction.

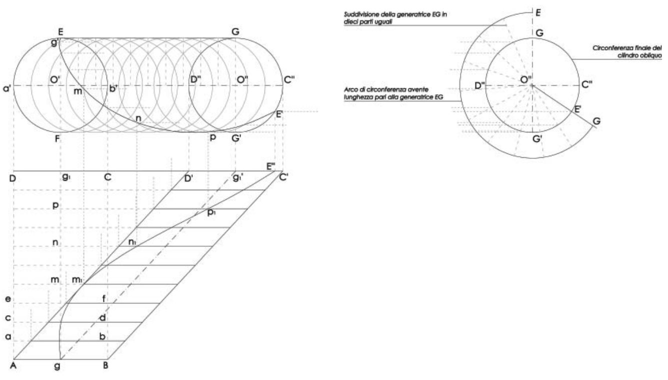


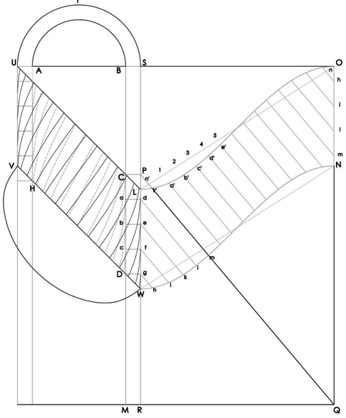


Figures 7 -8: Orthogonal equipment: design of the joint (left) and joint's projections; (*De Paola's drawing from Chaix* ,1890)

The treatise of *Joseph Alphonse Adhemar*, *Nouvelles Etudes de coupe des pierres*. *Traite théorique pratique des ponts biais*, (1856) as many of the nineteenth treatises, describes the various types of oblique bridges used in practice construction, devoting each chapter to a specific method of equipment, adding detailed descriptions of horizontal and transverse joints, and not neglecting the static part, with considerations about the balance of the vault.
So the matters and solutions about the theme of stereotomy have changed in engeneering research, with the consequent way of thinking the structure: from the stonecutting to the detailed matemathical calculation.
The mathematician of Mézières, *Jean Pierre Nicolas Hachette*, a disciple of *Monge*, dedicated a study on the geometric definition of equipment. Posted in 1854 in *The Annals of Bridges*, with the title of Descrip*tion of a new system for the construction of oblique vaults*, according Hachette, the systems used previously had a common flaw: The complexity of working that through his new method, called cycloidal, was resolved because both require drawings and representations of the faces very simple, and because the cut of stone was made from the face of flat leather production. From a mechanical way then, the balance is guaranteed because the joints efforts will send strings as if they were righteous.

In 1828, *Peter Nicholson* wrote the "*Popular and practical treatise on masonry and stone – cutting*" dealing mainly on the oblique helical, while *G. W. Buck* perfected the method proposed by his predecessors, arousing great interest to the problem, often cited by his contemporaries as one of the greatest experts on skew arches.





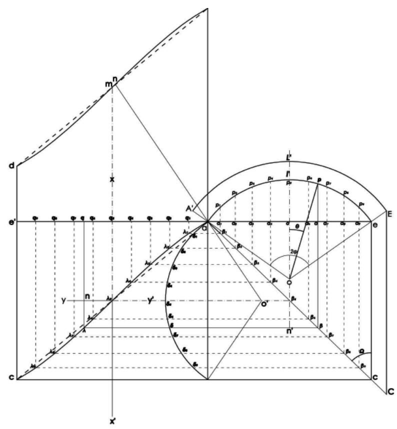


Figure 9: Elicoidal construction plan; (de Paola's drawing from *J. N. P. Hachette*)
Figure 10: Development of extrados; (de Paola's drawing from J. W. *Buck ,1839)*
Figure 11: Development with the oblique arch-dressing to a portion of semicircle and inclination than the horizontal line;(de Paola's drawing from J. W. Buck, 1839)

A practical and theoretical essay on oblique bridges, (1830) by *G. W. Buck* was the landmark of English literature on oblique bridges. The origin of ancient skew arches had already been reported by Buck in his writings, and his essay was intended "for the use of engineers and architects generally, but principally for those who have the immediate superintendance of public works, and who, being young men, it is to be hoped will not complain of its being too mathematical."
The essay was written as a result of the work on the London – Birmingham line. Surprising is that Buck used an Italian historical reference as a basis of discussion rather than one belonging to the French school: The bridge built in Turin, on the *Dora Riparia*, by *Carlo Bernardo Mosca* (1830), one of the few Italian buildings able to compare with French achievements. It is completely dedicated to the construction of arches with spiral oblique appeals, known as elicodal method, the formulas for determining the size of the angles, for working voussoirs, and practical examples of use of these formulas.
Francis Bashforth wrote *A practical Treatise on the construction of Oblique Bridges*, with spiral and equilibrated courses, divided into three chapters: *on the oblique bridge constructed with spiral courses*; *on construction of the bridge with oblique equilibrated courses*; *on the construction of oblique ends of direct arches*. Despite expressing a number of objections on helical spiral courses, he shows that it was the best method. The method expressed essentially agrees with Nicholson's and Buck's, even with many variations, especially in the details. Bashforth tried to translate all the theories previously described, using an easy language for those who should have to build oblique bridges in practice.
Jules Antoine René Maillard de la Gournerie became one of the protagonists of the international debate.
A descriptive geometry teacher at the *École Polytechnique* and the *Conservatoire des Arts et Métiers* of Paris, his interest in the mechanical aspects of *ponts biais* was confirmed by tests that were performed on a model of *arche biais* built at the Conservatoire des Art in 1874.
In 1851, de La Gournerie wrote a memoir to explain the rules to properly build an oblique bridge: the advantage of orthogonal joints, the best shapes of the arches, the relationship between oblique degree and stability. This theory was hardly confuted by the engeneer M Levy, according to whom the real solution for oblique

bridges was to cut orthogonally the joints, as well as the stone cutting tradition: a geometric construction not very safe with static behaviour.
The quarrel, involving many teachers in Europe, dealt with the thrusts' direction in the vault: parallel to the face planes, or depending by the equipment, or following the vertical lines of main curvation.

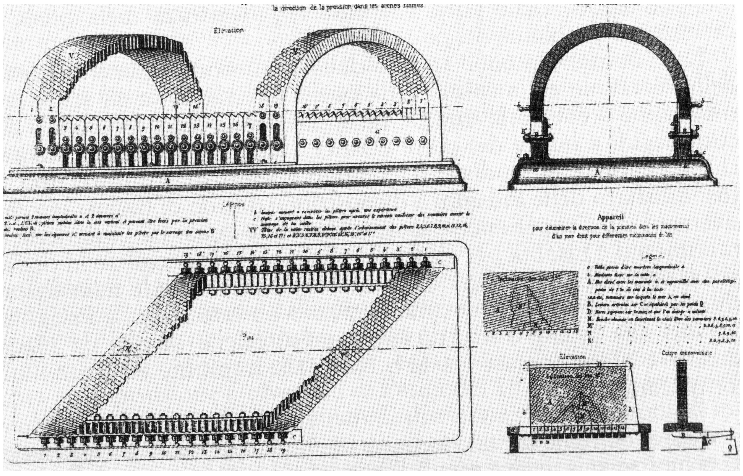


Figure 12: La Gournerie device to show thrusts direction in the vault; (Sakarovitch J., 1998)

According to La Gournerie, the thrusts were trasmitted following the parallel cut-lines of the faces, as he was able to study during the Haussmann's building plain: He prepared a device to experimentally determine the pressure direction in an oblique vault.
The question attracted the attention and criticism of other students, such as Emile Trèlat or Alfred D. Claye. The debate seemed to be concluded with the agreement that the orthogonal equipment was better able to give stability, but the helicoidal one was easier to build.
Antonio Bordoni, teacher of Physics at the University of Pavia in 1826 published a brief *Nota di stereotomia sopra i cunei dei ponti in isbieco*, where his interest in theoretical debate, and geometric debate in particular, is accompanied by practical recommendations for the selection of the shape of *voussoirs* forming the vault of these bridges.
Vittorio Baggi, a teacher of hydraulic engineering and road construction in the Royal School of Application Engineers in Turin, deals with the issue of bridges, deepening the study of those of the method with twisted ones and simplifying with the use of bricks.
G. Codazza, physicist and mathematician in Milan, was professor of descriptive geometry and Science of Construction of Machinery at the Royal University of Pavia from 1842: He wrote *Nozioni teorico-pratiche sul taglio delle pietre e sulle centine delle volte* in two volumes, the the first devoted exclusively to the geometric theory, the second an atlas including 29 double-page tables.
Antonio Cantalupi, a civil engineer, in *La costruzione dei ponti e dei viadotti*: trattato di architettura pratica of 1844, occupied for more examples of oblique bridges made in Italy: *Protche* on *Porrettana*, *Berman* in Udine, *Pontebba* in *Ospedaletto*, *Caimi* in *Boffalora* and *Ribecchi* on *Lambro,* expressing also in dealing with general, special interest in French solutions.
F. Colombani attended the *Ecole des Ponts et Chaussée* and gained experience in French bridges on the line Paris - Saint Germain, work directed by *Lamé* and *Clapeyron*. In *Sul taglio dei cunei dei ponti in sbieco*, Colombani prepared a device in which the stones safeguard the double standard of orthogonality and explains in detail the implementation of its system in case of circular or elliptical bridge. The method studied by Colombani fulfils the stability conditions and was supported by geometric demonstrations and mathematical analysis.
The *Annales des Ponts et Chaussées* became the place where present results and rules, especially in the field of stereotomy and mechanics, applied to buildings.
The matters were many: from constructive, linked to the choice of equipemment materials and techniques, to geometric shape of the bridge, its components and joints pose, to static ones, like determining pressure distribution within arches and supports.
While British engineers seemed more interested in convenient resolutions of practical problems, the French had a more speculative approach.
Through these rules, they also sought to ensure perfect stability, to avoid acute angles in *voussoirs* and maintain the perpendicularity of the radial joints respecting the intradox. Great attention was also given to study the direction of thrusts in the void, analysing the compression of the acute stones and the behaviour in key. To prevent this coming out from its position, it was necessary to make the oblique vault behaved like a right one, which channels the forces in perpendicular direction at the supports.
The authors were all agreed in that this could have been giving voussoirs precise geometric configuration, as the parallelism of heads with front arch of the vault, the perpendicularity between the longitudinal joints and the base front and finally the parallelism of joints with the direction of supports.

Thus orthogonal, helical, parallel, cycloidal and t perpendicular converging equipment arose; the orthogonal method appeared to be the most appropriate, especially in terms of structural static, as it was the only one that allowed to avoid the so-called thrusts in the void, due to adverse movements in the vault that in cases of extreme skew, could cause collapse. The elical method was instead more easy to plan and realise.

CONCLUSIONS

The theme of oblique arches became the main question not only for stereotomic speculation for half a century: The research stimulated the interest in different field and countries, arising a multidisciplinary study about a modern problem, through historical to new solutions. If at the beginning it was an engeneering matter to solve the practical problems of building, in short the skew arches became the point around whitch mathematicians, engineers and academics are involved to search for the best answer. Different solutions are found, according to the different geographic areas, schools, traditions and tecnologies, bringing different formal results. But important is also that this theme leads again attention on the stereotomic discipline when it seemed to set because of the changed cultural attitude of nineteenth century.

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